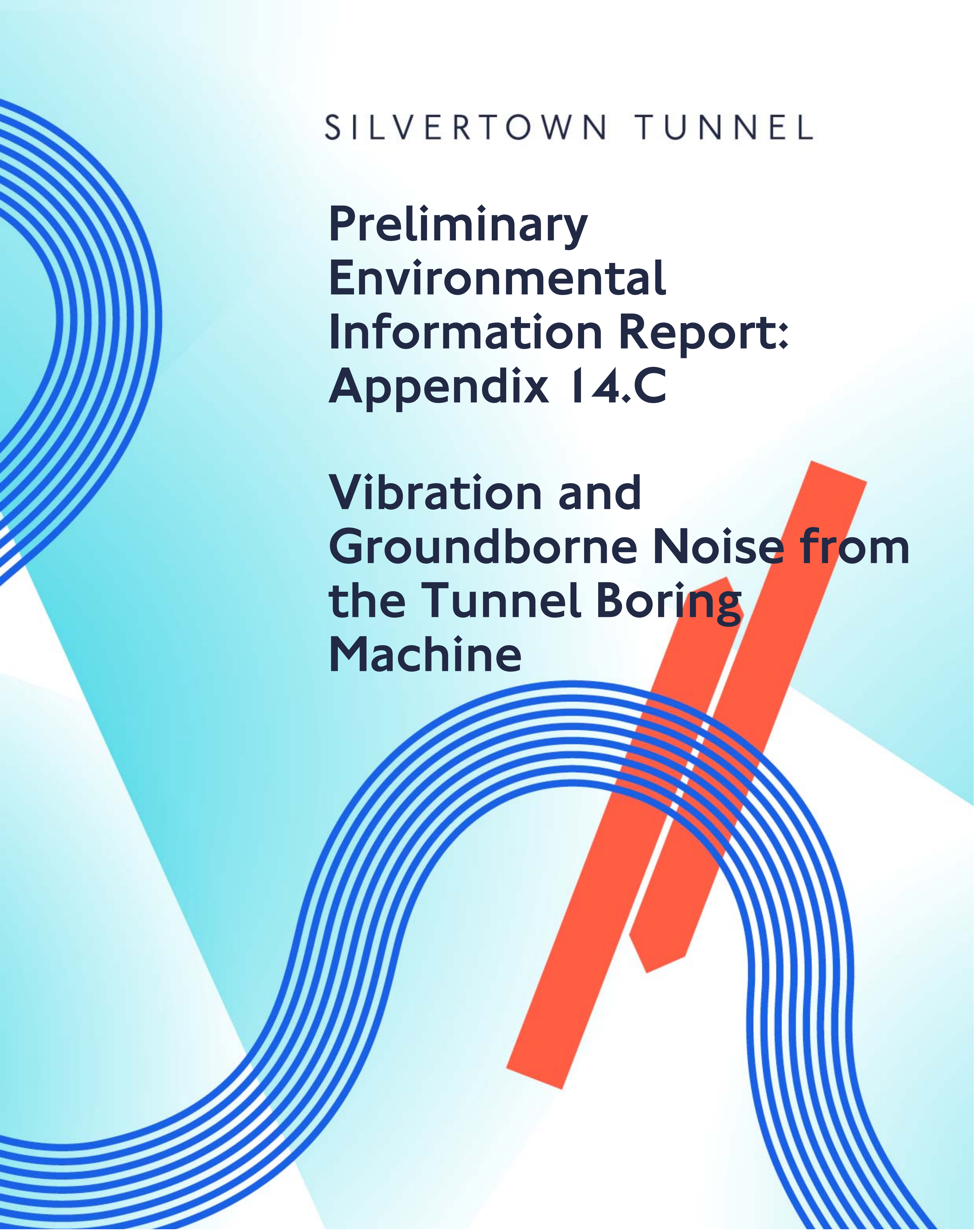
SILVERTOWN TUNNEL

# Preliminary Environmental Information Report: Appendix 14.C

# Vibration and Groundborne Noise from the Tunnel Boring Machine



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# Contents

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# 1. INTRODUCTION

1.1.1 This report sets out the findings of a study of the likely level of groundborne noise and vibration from the excavation of the proposed Silvertown Tunnels using a tunnel boring machine (TBM). The study has predicted the likely levels of groundborne noise and vibration in properties above the alignment during the passage of the TBM.

1.1.2 This report forms a technical appendix to the Preliminary Environmental Information Report (PEIR) for the Scheme, and should be read in conjunction with Chapter 14 Noise and Vibration.

1.1.3 Models were created to study the propagation of vibration from the tunnel face, with the TBM operating in the soils which are likely to occur along the alignment. The output of the modelling is an indication of likely ground vibration and associated groundborne noise at various depths and geological situations, as well as a prediction of the decay of vibration with distance.

1.1.4 Vibration and groundborne noise are different aspects of the same phenomenon. Vibration is oscillating movement of the ground or other solid material. This may cause sound to be radiated from vibrating surfaces. Vibration, if high enough in amplitude, may be perceived by the tactile sense. Re-radiated groundborne or structureborne noise is perceived by the sense of hearing at frequencies within the audible range. Vibration may be perceived at frequencies too low to be audible.

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# 2. METHODOLOGY

*Study Area and Receptors*

2.1.1 The Scheme is described in Chapter 4 of the PEIR, and an overview of the alignment is provided in Appendix 4.C. Potential vibration and noise from the TBM is limited to locations between each portal and the river foreshore. There are sensitive receptors near Silvertown Way in Newham, and in the vicinity of East Parkside and West Parkside in Greenwich.

2.1.2 It will also be necessary to consider the effects of vibration and underwater noise on fish and benthic species and this will be the subject of a further modelling exercise.

*Significance Criteria*

2.1.3 Vibration is assessed in the UK by means of the Vibration Dose Value (VDV) which is defined in to BS 6472-1:2008 “Guide to evaluation of human exposure to vibration in buildings”. There is no equivalent standard for groundborne noise, but the convention has been established that groundborne noise is assessed in terms of the maximum A-weighted sound level using the index $L_{pASmax}$. VDV is based on weighted acceleration and is dependent on the number and duration of events. However, if vibration velocity does not exceed a continuous rms level of 0.5mm/s throughout an 8-hour night it will not be likely to exceed $0.1ms^{-1.75}$ VDV.

2.1.4 It is anticipated that the tunnel boring machine would travel at approximately 30-50 metres per day, which needs to be taken into account in determining significance criteria. However, it is recognised that there is no mitigation available for groundborne noise, other than temporary re-housing.

2.1.5 Crossrail did not have a significance threshold for TBM groundborne noise, it was explicitly excluded from the sources covered by the 40 dB $L_{Amax}$ figure. The passage of the TBM on Crossrail went largely without incident, and was not much above 40 dBA at 14m depth (based on monitoring at Sussex Gardens) and less at greater depths.

2.1.6 The Silvertown Tunnel Scheme TBM will be much larger, with a face area some three times that of Crossrail. As far as noise policy thresholds are concerned, it is considered that Significant Observed Adverse Effect Level

(SOAEL) is 45 dB LAsmax and the Lowest Observed Adverse Effect Level (LOAEL) at 35 $L_{Asmax}$. Table 2-1 sets out the project's criteria for vibration and groundborne noise.

**Table 2-1 Assessment thresholds for groundborne noise and vibration**

| | | | |
|---|---|---|---|
| Ground-borne noise | Lowest Observed Adverse Effect Level | $L_{pASmax}$ dB | 35 |
| | Significant Observed Adverse Effect Level | $L_{pASmax}$ dB | 45 |
| Vibration | Lowest Observed Adverse Effect Level | $VDV_{day}$ $m/s^{-1.75}$ | 0.2 |
| | | $VDV_{night}$ $m/s^{-1.75}$ | 0.1 |
| | Significant Observed Adverse Effect Level | $VDV_{day}$ $m/s^{-1.75}$ | 0.8 |
| | | $VDV_{night}$ $m/s^{-1.75}$ | 0.4 |

*Experience from Other Schemes*

2.1.7 Tunnel boring machines have been used for tunnel excavation in London since the construction of the Jubilee Line Extension to replace the former shield-and-backhoe method. Projects have included the Docklands Light Railway extension to Greenwich, Crossrail and the Channel Tunnel Rail Link (High Speed 1). In the environmental statements for those projects it was considered that there would be no significant effect because although the passage of the machines would be audible the duration would be limited to a few days. Some complaints were received with respect to tunnelling for the Channel Tunnel Rail Link operating in harder soil types.

2.1.8 During the construction of the Dublin Port Tunnel, a road tunnel in which a much larger TBM was used to excavated limestone and glacial till, there were strong complaints about noise and vibration which occurred at much higher levels than was the case for the rail tunnels in London. This led to greater attention being paid to noise and vibration from TBM working in subsequent projects, such as Dublin Metro North. Monitoring of groundborne noise and vibration was carried out during TBM working on the Crossrail Project. Detailed predictions of vibration and groundborne noise were not carried out prior to the Dublin Port Tunnel, and since then they have been done for Metro North, Corrib onshore pipeline and Silvertown Tunnel. Groundborne noise and vibration was specifically considered in the HS2 Environmental Statement with the conclusions that there would be no significant adverse effects.

2.1.9 The factors which influence the generation and propagation of vibration and groundborne noise from TBMs are primarily the amount of energy required to cut the soil (or rock in the case of Dublin) and the propagation characteristics of the soil. Rotational speed, cutter head type and face pressure have a much smaller effect. The energy requirement is a function of the tunnel diameter and the operating characteristics of the machine. Each bore of the proposed Silvertown Tunnel is likely to be approximately 12.5m diameter. The diameter of the Jubilee Line Extension TBM face was 4.9m, Dublin Port Tunnel was 11.8m, Crossrail 7.1m and High Speed 1 London tunnels were 8.11m diameter.

2.1.10 Soil type is a major influence, with London Clay being soft enough for the main noise from the TBM to be machinery on the TBM. At the other extreme, excavating through rock generates a large amount of noise and vibration due to the cutting effect itself. The other schemes discussed above were located in a variety of lithologies. Dublin is carboniferous limestone below glacial till. The tunnels in London are in London Clay, Gravel, Lambeth Beds, Chalk and Thanet sands. The Silvertown Tunnel will be bored through clay, gravel and sand.

2.1.11 The tunnels referred to above were bored using earth pressure balance machines. In some cases slurry machines are used, but it is possible that an EPB machine will be used for the Silvertown Tunnel construction, therefore this has been assumed for the assessment as a reasonable worst case assumption. Slurry machines are mainly used in highly permeable unstable terrain where vibration generated by the cutting action at the face would be very low.

### *Prediction Of Vibration And Noise From Tunnel Boring Machines*

2.1.12 The prediction of vibration and groundborne noise from tunnel boring machines has to begin with measured field data obtained on other projects, principally Crossrail, which is used to calibrate the output of a model for predicting the spatial spread of the vibration (which in turn may also cause groundborne noise). In stiff or hard soils the source is concentrated at the cutter face. In soft soil, groundborne noise may be radiated from the entire length of the TBM, which can reach lengths of 100m or so (see cutaway illustration in Figure 1).

**Figure 1 Cutaway view of a large diameter tunnel boring machine (Herrenknecht)**

*Numerical Modelling*

2.1.13 The predictions were carried out using the Rupert Taylor Finite Difference Time Domain model *FINDWAVE*®.

2.1.14 The model used for this study predicts, in the time domain, the three-dimensional vibration velocity of the tunnel face and surrounding lithology. The time-domain results are transformed into the frequency domain to give 1/3 octave frequency spectra, and overall sound levels in dB(A) and vibration units.

2.1.15 The model has been calibrated by using the model to predict vibration from the Crossrail TBM, and back-fitting the results from field measurements obtained during the tunnel drive.

2.1.16 The approach has been to set up a group of generic models in a selection of soil conditions and produce cross-sectional plots of vectored soil velocity from which, subject to the application of transfer functions to buildings, ground surface predictions can been made.

2.1.17 *FINDWAVE*® is a finite difference time-domain numerical model for computing the propagation of waves in elastic media. Full details of the model are given in Appendix I. The excitation (source of vibration) is provided from a random array of impulses applied to the tunnel face. The model predicts, in the time domain, the dynamic behaviour the medium surrounding the tunnel face.

2.1.18 The model has a cell size of 100mm in the lateral and vertical directions, and 100mm in the longitudinal direction (along the tunnel). A time step of 1/131072 seconds was used. The model was run for a time period of 1 second. Output from the model consists of time series of the velocity of transverse and longitudinal sections through the model, which are subjected to frequency transformation and expressed as 1/3 octave band spectra. A model with a cell size of 250mm x 250mm was also used to study the effect of distance out to 80m.

2.1.19 At this stage, not enough detail is known about the structure and foundations of potentially affected sensitive buildings to enable detailed predictions to be made for specific receptors. A high resolution model was set up to look at the effect of the presence of a building (a brick-built house on a slab foundation) on the levels of airborne sound likely to occur within rooms due to ground-transmitted vibration at acoustic frequencies.

### *Modelling Assumptions Used*

2.1.20 This geotechnical data assumed in the modelling are shown in Table 2-2 and Table 2-3 based on the Atkins Ground Investigation Report 2015.

**Table 2-2 Typical strata boundaries on the northern side of the Thames**

| **Formation** | **Typical Soil Description** | **Top (mAOD)** | | **Bottom (mAOD)** | | **Top (mBGL)** | | **Bottom (mBGL )** | | **Thickness (m)** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max |
| Made Ground | Brick rubble, ash, sand | 2.13 | 5.72 | -0.91 | 2.57 | 0.0 | 0.0 | 0.91 | 6.2 | 0.91 | 6.2 |
| Alluvium | Silty CLAY with pockets of peat | -0.91 | 2.57 | -3.95 | -0.48 | 0.91 | 6.2 | 3.66 | 9.45 | 1.22 | 4.5 |
| River Terrace Deposits | Silty Sandy GRAVEL | -3.95 | -0.48 | -10.96 | -6.88 | 3.66 | 9.45 | 10.36 | 16.0 | 5.95 | 8.38 |
| London Clay Formation | Stiff silty CLAY | -10.96 | -6.88 | -16.93 | -11.86 | 11.58 | 16.0 | 14.02 | 22.65 | 0.9 | 6.8 |
| Harwich Formation | Dense black Pebbles (GRAVEL) | -16.93 | -14.48 | -22.76 | -15.39 | -17.53 | 22.65 | 18.44 | 28.48 | 1.02 | 5.83 |

| **Formation** | **Typical Soil Description** | **Top (mAOD)** | | **Bottom (mAOD)** | | **Top (mBGL)** | | **Bottom (mBGL)** | | **Thickness (m)** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Lambeth Group | Very dense pale green / blue SAND | -22.76 | -6.88 | -35.26 | -18.8 | 10.36 | 28.48 | 24.3 | 40.6 | 8.9 | 14.8 |
| Upnor Formation | Silty fine to medium SAND | -35.26 | - | -37.41 | - | 40.6 | - | 42.75 | - | 2.15 | - |
| Thanet Formation | Very dense silty fine SAND | -37.41 | -18.8 | -45.9 | -29.5 | 24.3 | 42.75 | 35 | 49.38 | 10.7 | 12.5 |
| Chalk | N/A | -45.9 | - | N/A | N/A | 49.38 | - | N/A | N/A | N/A | N/A |

**Table 2-3 Typical strata boundaries on the southern side of the Thames**

| **Formation** | **Typical Soil Description** | **Top (mAOD)** | | **Bottom (mAOD)** | | **Top (mBGL)** | | **Bottom (mBGL)** | | **Thickness (m)** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max |

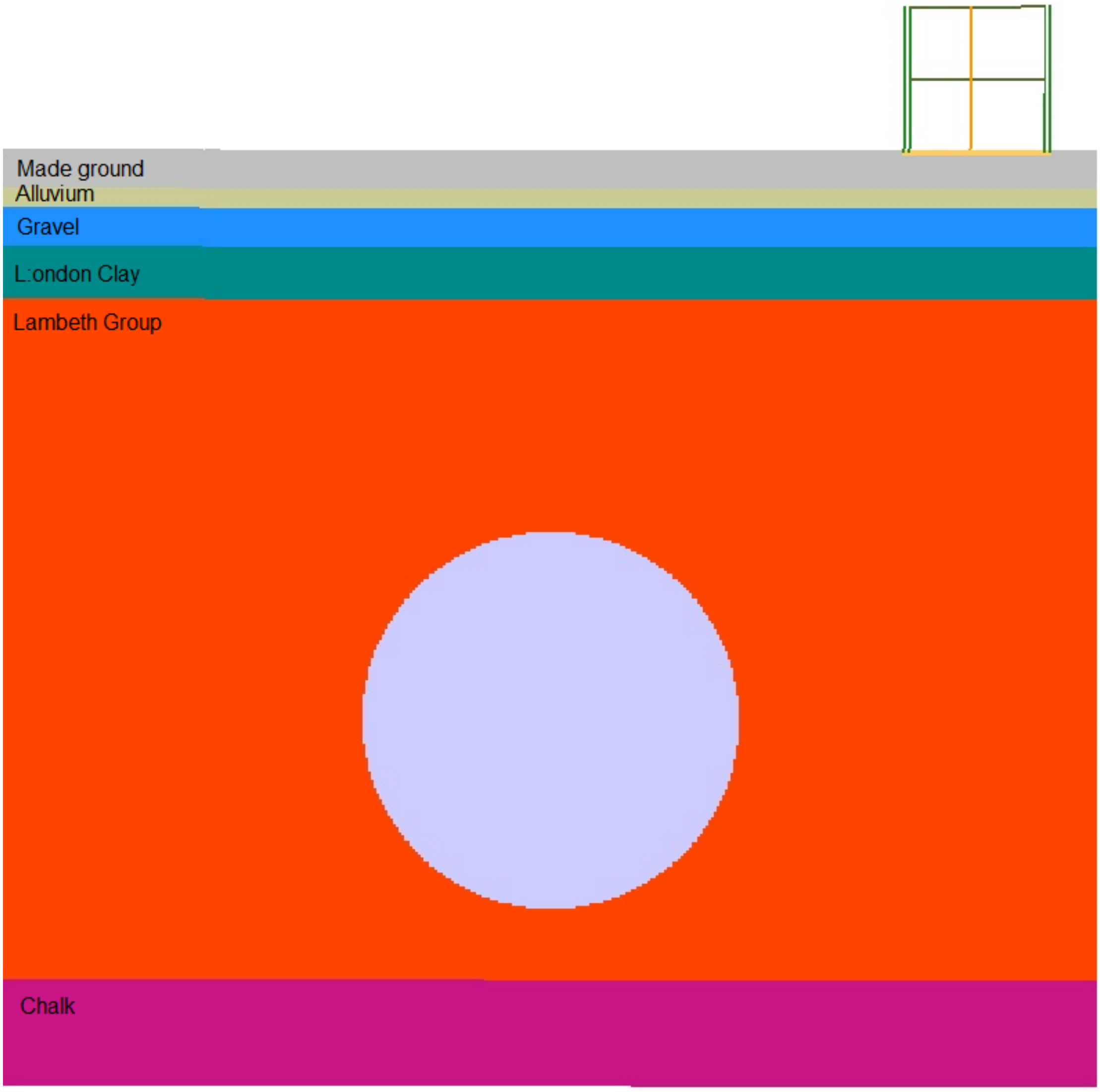
| **Formation** | **Typical Soil Description** | **Top (mAOD)** | | **Bottom (mAOD)** | | **Top (mBGL)** | | **Bottom (mBGL)** | | **Thickness (m)** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Made Ground | Brick rubble, ash, sand | 1.35 | 5.28 | -9.22 | 1.76 | 0.0 | 0.0 | 1.0 | 14.50 | 1.0 | 14.50 |
| Alluvium | Silty CLAY | -3.23 | 1.76 | -5.95 | -1.1 | 1.0 | 8.1 | 3.2 | 10.3 | 1.45 | 7.7 |
| River Terrace Deposits | Silty Sandy GRAVEL | -5.84 | -1.1 | -8.74 | -4.43 | 3.2 | 10.3 | 6.6 | 13.9 | 1.6 | 4.4 |
| London Clay Formation | Stiff CLAY | -9.22 | -4.43 | -22.3 | -16.54 | 6.6 | 14.5 | 18 | 26.04 | 9 | 17.9 (P) |
| Harwich Formation | Very dense GRAVEL | -20.76 | -19.48 | -25.48 | -20.54 | 14.5 | 26.04 | 15.02 | 30.64 | 0.52 | 5.17 |
| Lambeth Group | Very dense pale green / blue SAND | -25.48 | -20.15 | -40.08 | -27.83 | 15.02 | 30.64 | 30.85 | 45.24 | 5 | 15.83 |
| Upnor | Silty fine to | -38.97 | -36.37 | -40.47 | -39.33 | 30.85 | 44.25 | 33.81 | 45.25 | 1.5 | 2.96 |

| **Formation** | **Typical Soil Description** | **Top (mAOD)** | | **Bottom (mAOD)** | | **Top (mBGL)** | | **Bottom (mBGL)** | | **Thickness (m)** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Formation | medium | | | | | | | | | | |
| | SAND | | | | | | | | | | |
| Thanet | Very | -40.47 | -39.33 | -52.52 | -50.44 | 33.81 | 45.75 | 47.0 | 56.88 | 10.02 | 13.19 |
| Formation | siltySAND | | | | | | | | | | |
| Chalk | N/A | -52.52 | -50.44 | N/A | N/A | 47.0 | 56.88 | N/A | N/A | N/A | N/A |

2.1.21 The tunnel behind the TBM was assumed to be lined with concrete with properties as given below, along with the properties assigned to the lithology.

2.1.22 Figure 2 is a typical cross section through the model showing example soil layers and the building included in the model to study the transfer of vibration to internal groundborne noise.

**Figure 2 Typical cross section through the model showing soil layers**



2.1.23 The modulus assumptions used are relevant to the extremely small strains involved in groundborne noise and vibration, and are not necessarily the same as those used for civil engineering purposes. The property D is the compressive modulus, given by:

$$D=2G(1-\sigma)/(1-2\sigma)$$

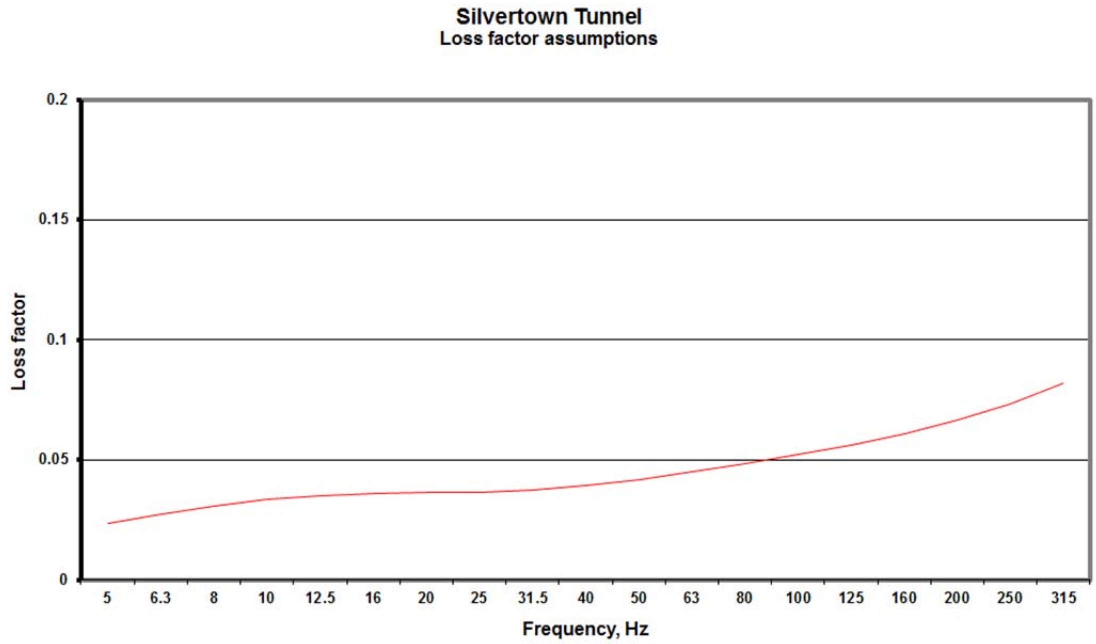
where $\sigma$ is Poisson's ratio and G is shear Modulus.

**Table 2-4 Soil properties assumed in the model**

| Material | Shear Modulus, $G$, $GNm^{-2}$ | Compression Modulus, $D$, $GNm^{-2}$ | Density, $\rho$ $kg/m^3$ |
|---|---|---|---|
| Air | 0 | 0.00014 | 1.18 |
| Concrete | 11.6 | 31.33 | 2400 |
| Made Ground | 0.068 | 0.266 | 1500 |
| Alluvium | 0.068 | 0.266 | 1500 |
| Gravel | 0.027 | 1.035 | 2000 |
| London Clay | 0.0735 | 4.41 | 1700 |
| Lambeth Group | 0.58 | 5.9 | 2100 |
| Chalk | 0.3 | 0.845 | 1950 |

2.1.24 The loss factor assumed in each case was frequency dependent as shown in Figure 3. It should be noted that this represents solely material damping within the soil. Additional losses occur in propagation through layered ground, and these are separately computed within the model. Published literature sometimes includes all loss effects within the loss factor, leading to a figure higher than that given in Figure 3.

**Figure 3 Material damping - loss factor assumed**



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# 3. RESULTS

3.1.1 Groundborne noise is commonly estimated by prediction of the vibration of a surface such as a floor and converting that to what has become known as “pseudo noise level” using a formula[1] originally proposed by Kurzweil:

$$L_p=L_v-27 \text{ dB}$$

3.1.2 where $L_p$ is the airborne sound pressure level and $L_v$ is the velocity level in dB re 1 nanometre per second. There are some indications[2] that this may be an overestimate in some cases, by an amount of the order of 5 dB.

3.1.3 The first model run was carried out to consider the relationship between ground vibration and sound pressure level inside an example residential building – a brick structure on a concrete raft foundation. Figure 4 shows the actual A-weighted sound pressure level computed for a height of 1.25m above ground. Of principal interest is the plot inside the rooms of the example building (the rectangular feature to the right of the figure). Figure 5 shows pseudo noise level computed by the subtraction of 27 dB from the ground surface vertical velocity level (the highest zone plotted in these figures is 40-41 dBA). It shows that if the pseudo noise level approach is used based on free vibration of the ground surface, the prediction would be a maximum of 41 dB $L_{ASmax}$ although the level is highly dependent on room dimensions and position within the room, with highest levels near the walls and in the corners of the rooms. Bearing in mind that fact that bedheads are usually close to walls, these areas are highly relevant. Figure 5 shows pseudo noise of 44-45 dB, suggesting that the Kurzweil formula may well be a worst case.

3.1.4 Given the sensitivity of the airborne noise predictions to room dimensions and location, it is considered appropriate to use pseudo noise level in this study generally.

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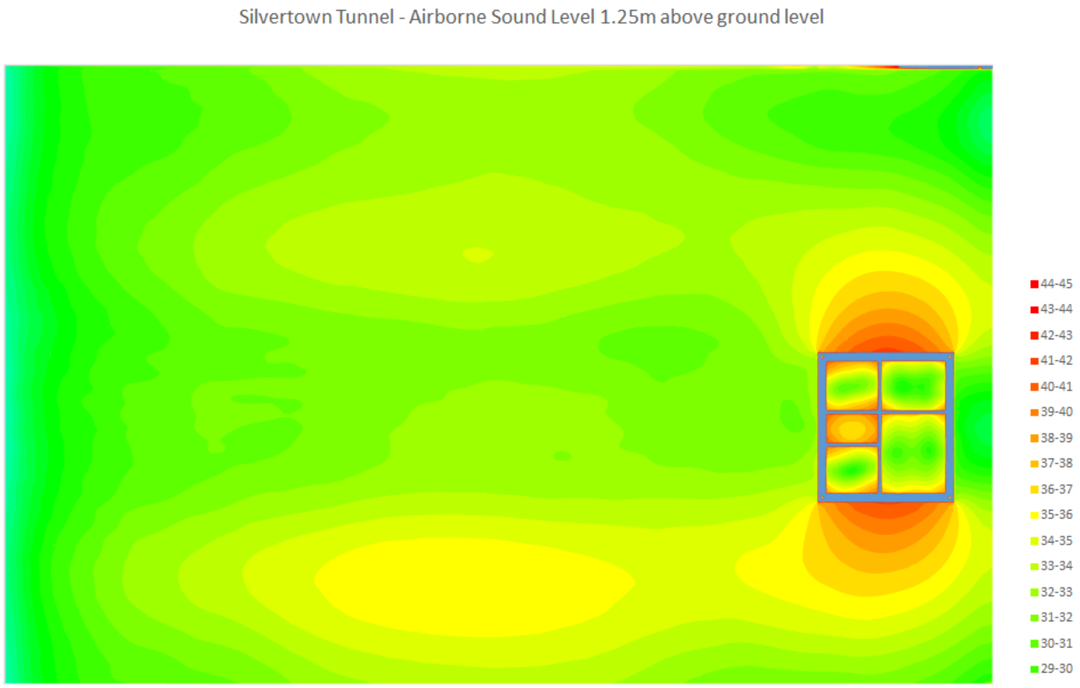
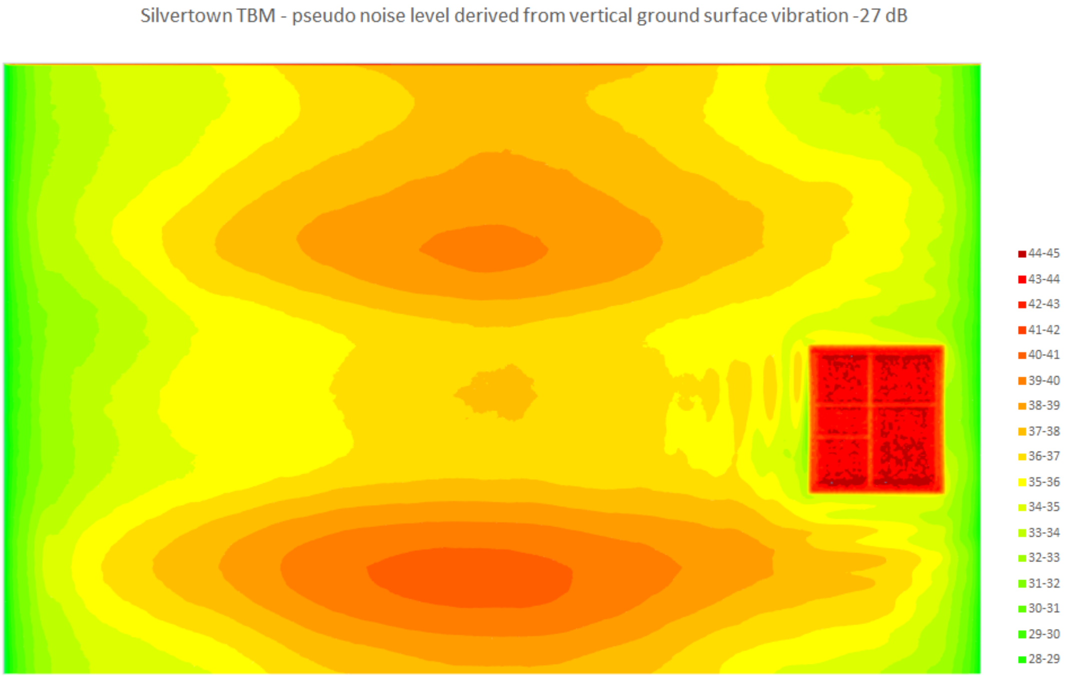
[1] Kurzweil, L.G. Ground-borne noise and vibration from underground rail systems, Journal of Sound and Vibration (1979) 66(3), 363-370

[2] Association of Noise Consultants, Measurement and Assess of Groundborne Noise and Vibration, 2012

## Figure 4 Actual airborne sound level



## Figure 5 Pseudo noise level derived from ground surface vibration



3.1.5 In order to study the effect of distance out to the order of 80m to the side of the tunnel a coarser grained model was used with a cell size of 250mm. The corresponding spectra are shown in Figure 8 and 9.

3.1.6 The effect of distance on both the spectrum the overall level of groundborne noise and the rms vibration velocity us shown in Figures 10 to 13

**Figure 6 Pseudo noise level to 80m distance – 14m ground cover**

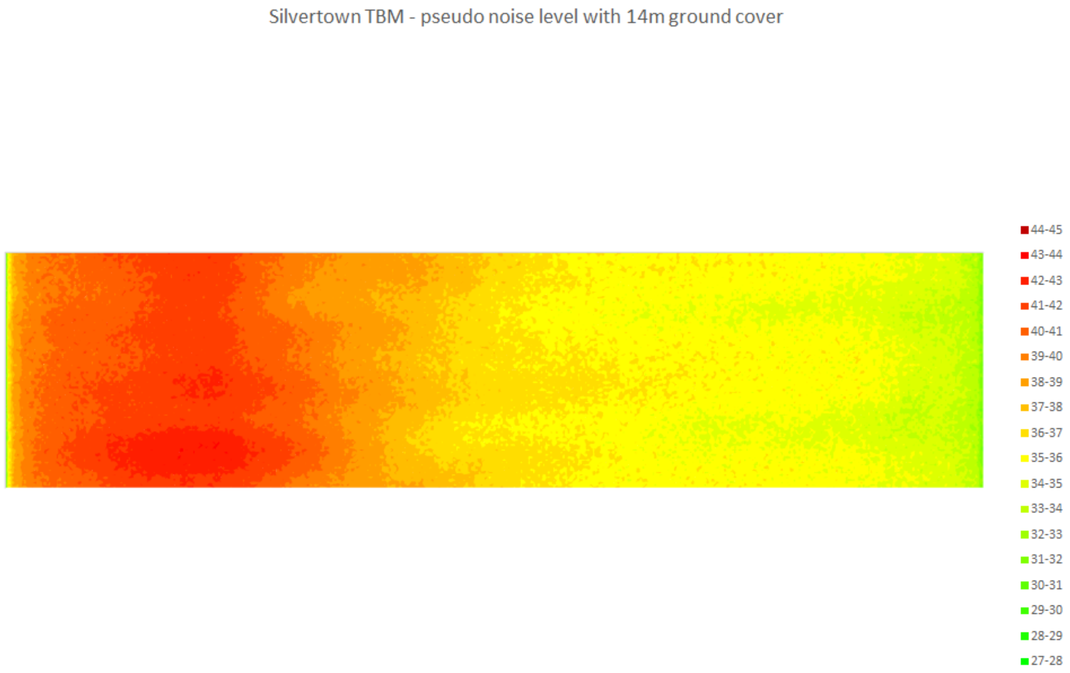


Figure 7 Pseudo noise level to 80m distance – 24m ground cover

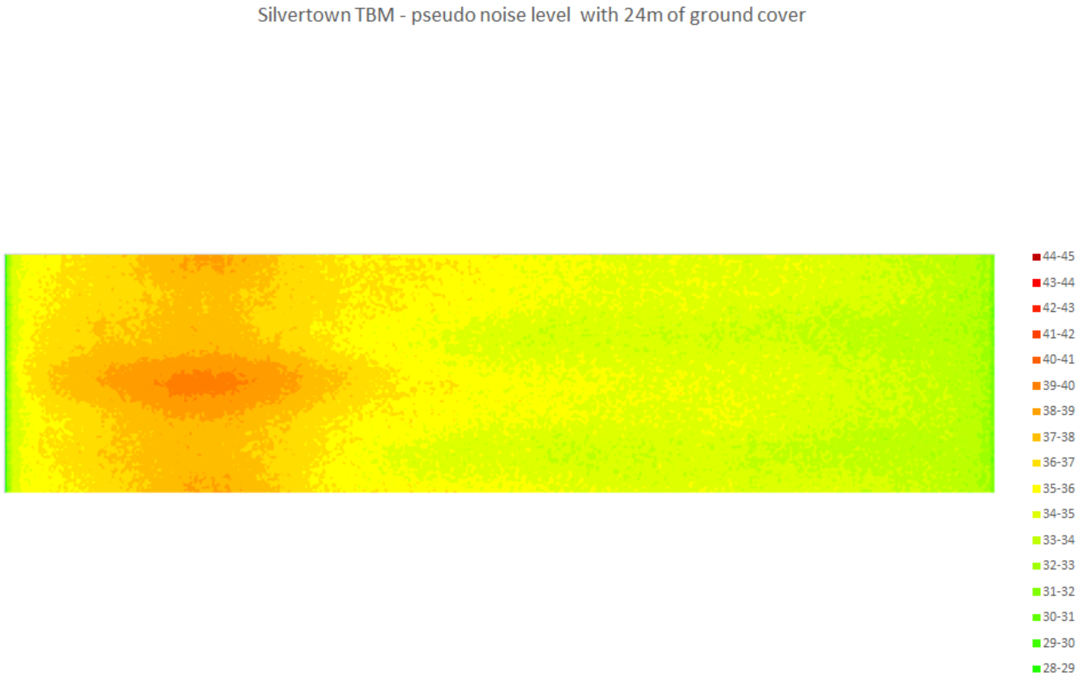


Figure 8 Spectrum above tunnel 14m ground cover

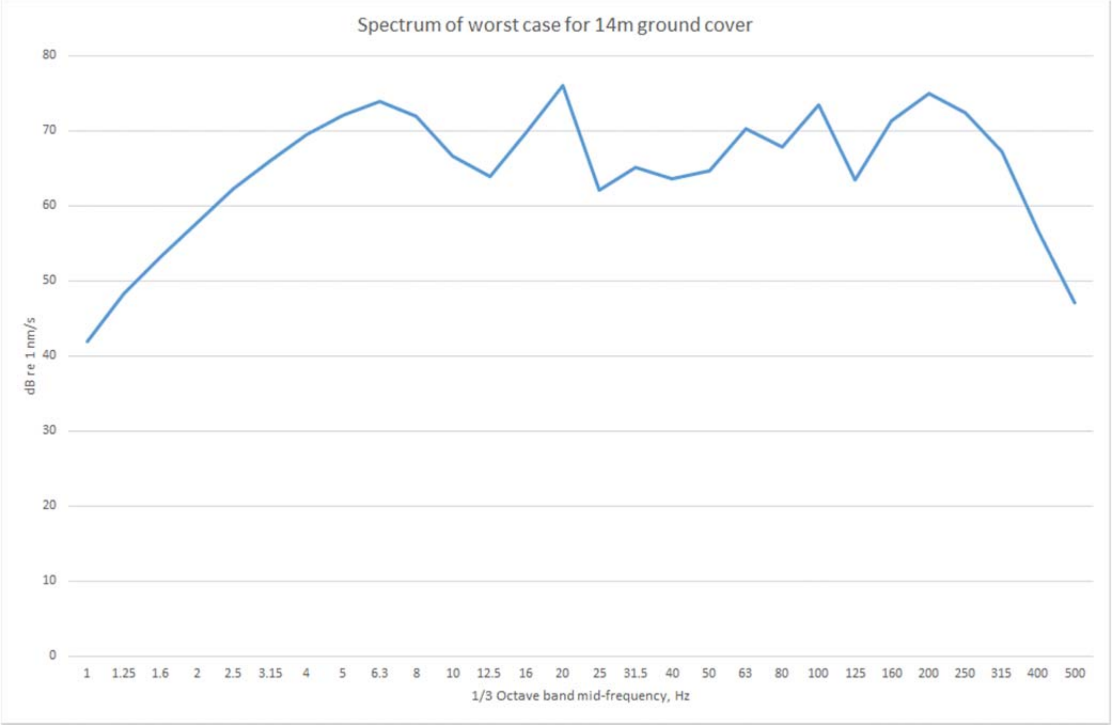


**Figure 9 Spectrum above tunnel 24m ground cover**

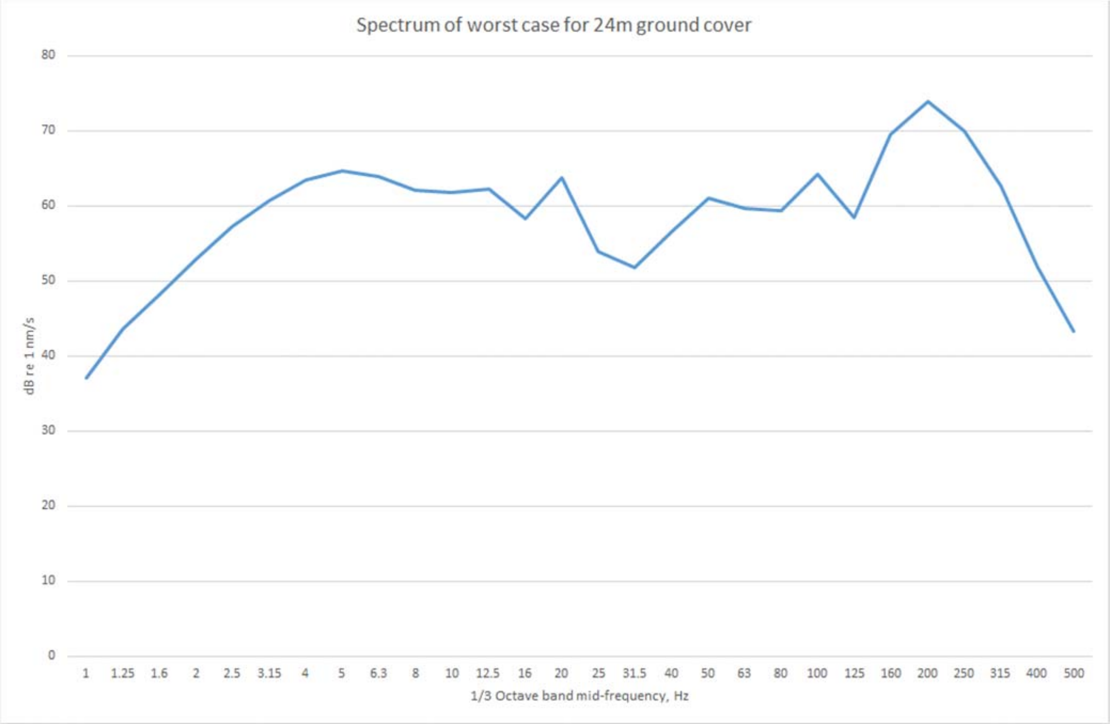
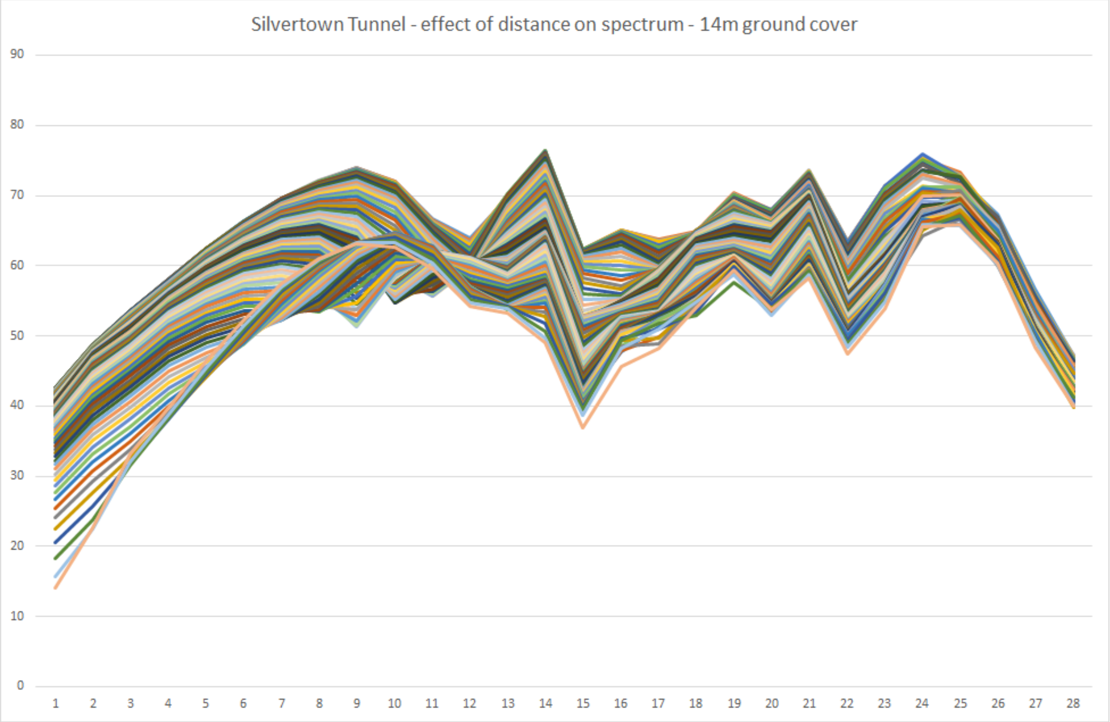
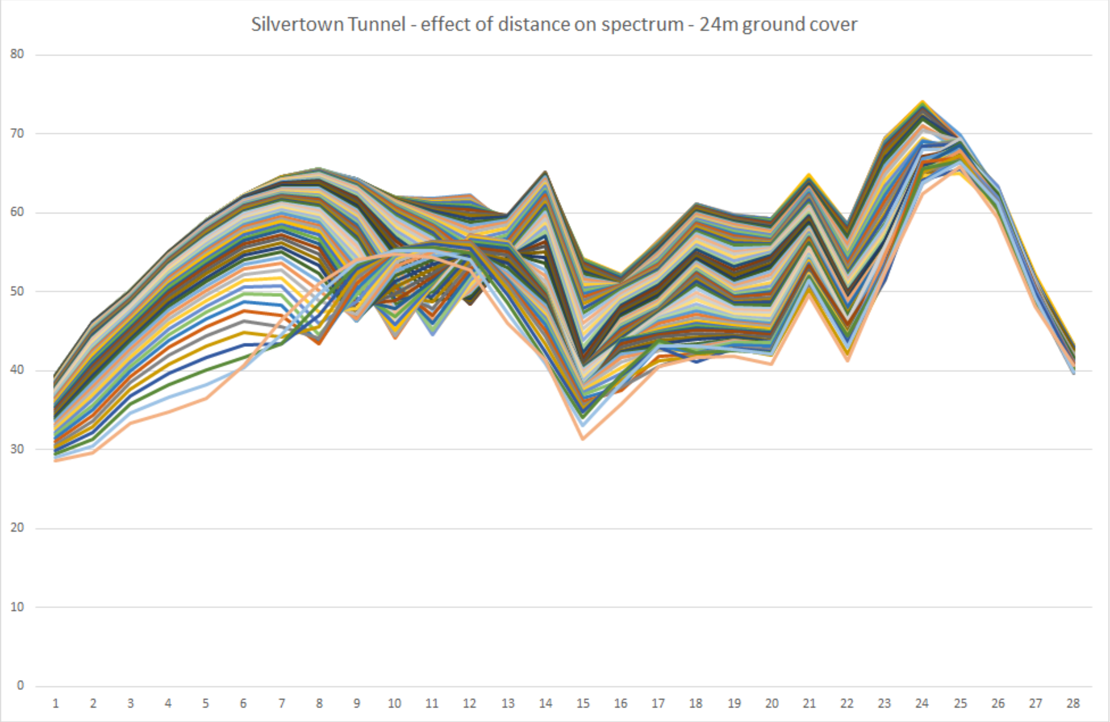


**Figure 10 Effect of distance on spectrum 14m ground cover**



### Figure 11 Effect of distance on spectrum 14m ground cover



### Figure 12 Effect of distance on LASmax

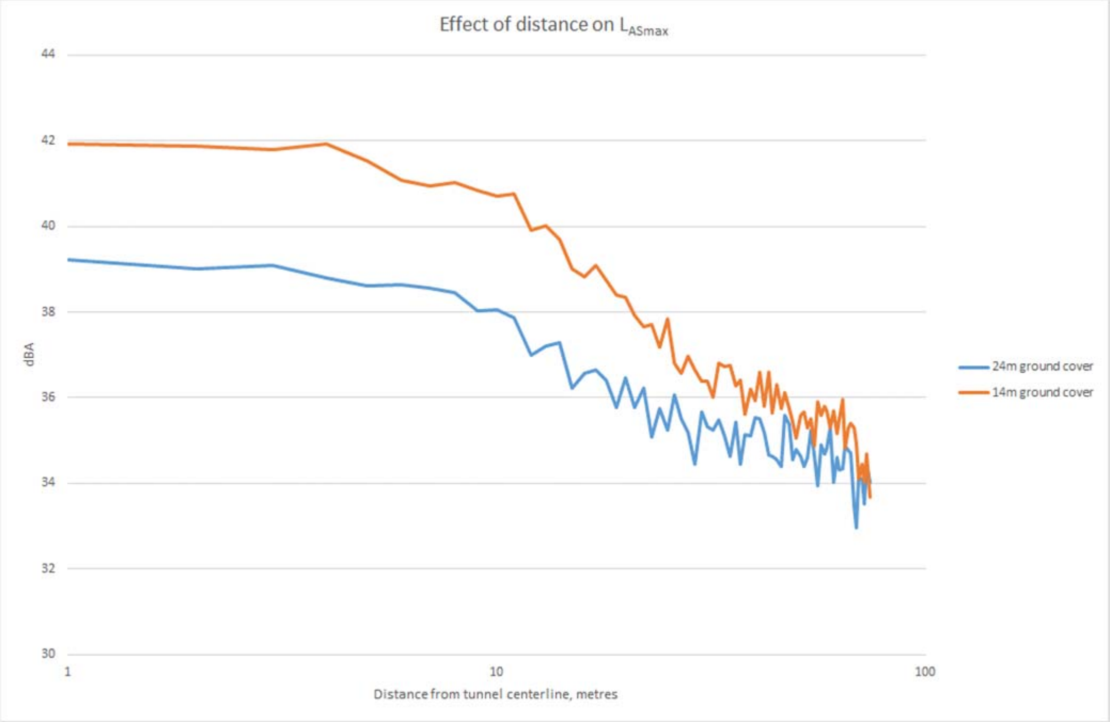
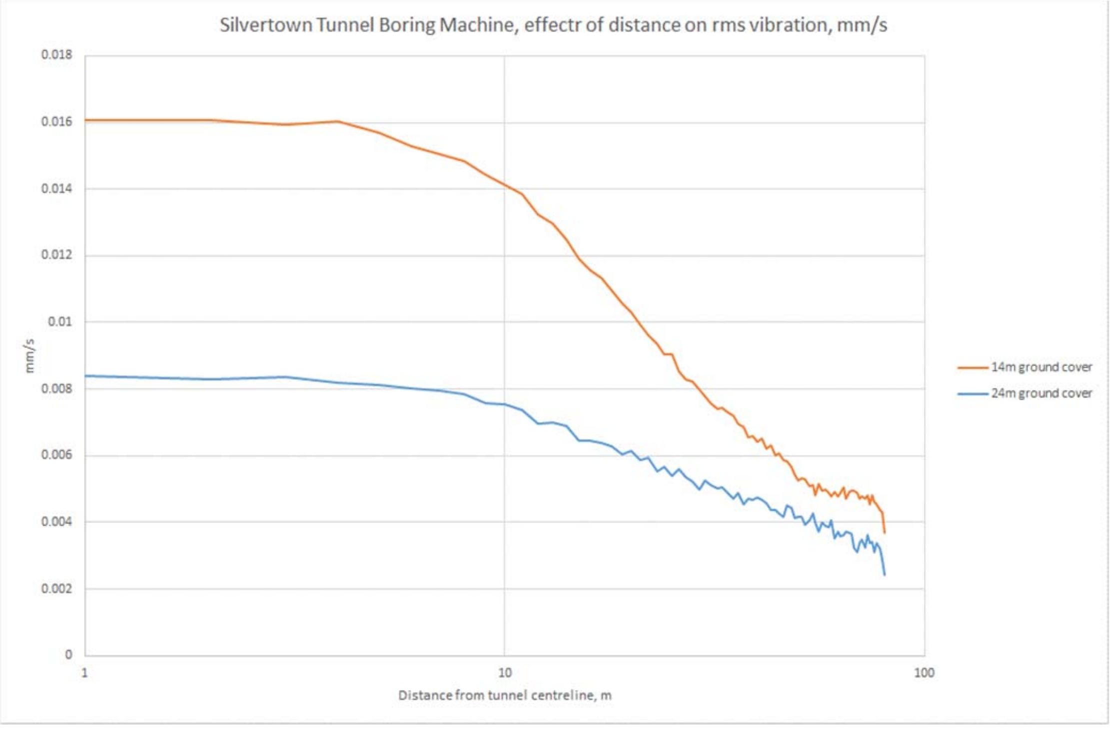


**Figure 13 Effect of distance on vibration, mm/s**



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# 4. DISCUSSION

4.1.1 According to the calculations, the groundborne noise results for the Silvertown Tunnel TBM drive are about 5 dB greater in level than was the case for the Crossrail tunnel drive. This is partly due to the increased face diameter, which for all other things unchanged would be expected to cause an increase of 5 dB. Boring in the Lambeth Group beds is likely to have a greater effect than boring in the London Clay but this is offset by the effect of the layered ground.

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# 5. CONCLUSIONS

5.1.1 The study has shown that the level of groundborne noise is unlikely to exceed the Significant Adverse Effect level even if there were sensitive receptors directly above the tunnel. Actual sensitive receptors are located to the side of the tunnel where the levels will be lower and the Lowest Observed Adverse effect level will be reached at about 75m to the side of the tunnel regardless of tunnel depth.

5.1.2 The Lowest Observed Adverse Effect Level for vibration is unlikely to be exceeded.

5.1.3 There is uncertainty associated with the predictions in that the soil properties have been derived from library data for the characteristics of the strata found in the ground investigation and not from field measurements. The method used for deriving groundborne noise levels from predicted ground vibration is, however, conservative.

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# Appendix A THE *FINDWAVE*® MODEL

## A.1 INTRODUCTION

A.1.1 The wave equation in differential form is as follows:

(1)

$$\mu\left(\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}+\frac{\partial^2\xi}{\partial z^2}\right)+(\lambda+\mu)\left(\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\eta}{\partial x\partial y}+\frac{\partial^2\zeta}{\partial x\partial z}\right)=\rho\frac{\partial^2\xi}{\partial t^2}$$

A.1.2 for the *x* axis, with corresponding equations for the *y* and *z* axes, where *x, y, z* and $\xi, \eta, \zeta$ are displacements in three orthogonal axes; $\lambda$ and $\mu$ are Lamé constants and $\rho$ is the density. The Lamé constant $\mu$ is also known as the shear modulus, *G*. The Lamé constant $\lambda$ is also known as the coefficient of dilatation and is given by:

$$\lambda=\frac{2\sigma G}{(1-2\sigma)}$$

where $\sigma$ is Poisson's ratio.

A.1.3 Equation (1) can be stated in finite difference form by replacing the differential operator with the approximation:

$$(2)\quad \frac{\partial\xi}{\partial x}\approx (x[i][j][k]\ -\ x[i-1]\ [j][k])/\ \Delta x$$

For $\Delta x \rightarrow 0$ these two forms are identical.

A.1.4 For a homogeneous, isotropic medium with a finite value for Δx, Δy and Δz, elastic wave propagation can be computed using the finite difference substitution of equation (2)

A.1.5 Effectively, the process is as follows, for each axis, i, j and k. The example given is for axis i. Each point p(i,j,k) lies at the corner of a rectangular cell and is assigned a mass equal to one eighth of the sum of the eight contiguous cells as well as a displacement and velocity. The displacement and velocity is interpolated for each intermediate "virtual" point p(i+d,i+d,k+d) where d=0 or 0.5.

1) Compute pressure gradient
2) Compute shear force gradient
3) Accelerate p(i,j,k) by $\Delta v = F/\rho\, \Delta t$ where $F$ is the sum of the force 1 & 2 and $\rho$ is the density assigned to the point and $v$ is the point velocity.
4) Displace p(i,j,k) by $\Delta x = \Delta v * \Delta t$ where $x$ is the point displacement and $t$ is one time step.
5) Repeat from step 1

A.1.6 The geometric part of wave propagation is completely represented by this process. Further terms are required to represent damping. Of several possible terms, the inclusion of a coefficient by which the velocity is multiplied produces a loss factor which decreases within increasing frequency (and gives rise to an excess attenuation per unit distance which is independent of frequency). A viscous damping term can be used, by including a force proportional to acceleration multiplied by a coefficient. However, many materials exhibit hysteretic damping, or damping with other types of frequency dependence. To model these effects it is necessary to include an algorithm which implements Boltzmann’s strain history method where:

$$s(t) = D_1\varepsilon(t) - \int_0^{\infty} \varepsilon(t-\Delta t)\varphi(\Delta t)\,d(\Delta t)$$

where $\varphi(\Delta t) = \frac{D_2}{\tau} e^{-\Delta t/\tau}$ is an after-effect function, $D_2$ is a constant and $\tau$ is a relaxation time. $D_1$ is a modulus, *s(t)* is stress and $\varepsilon(t)$ is strain. By combining several after-effect functions with different values of $D_2$ and $\tau$ any relationship between loss factor and frequency may be represented. Note that in the frequency domain the integral has a real and imaginary part, with the result that the value of the modulus is reduced by the inclusion of the relaxation terms. Depending on the choice of the constants and relaxation times, the stiffness of a resilient element will be frequency-dependent, and the value of $D_1$ must be adjusted at the same time that $D_2$ and $\tau$ are selected to give the required dynamic stiffness. This method has been implemented in the version of *FINDWAVE®* used for this study.

## A.2 BOUNDARIES

A.2.1 For modelling finite objects fully surrounded by space, the boundaries can be represented by assigning zero-valued elastic moduli to the space provided that the acoustic load of the air in an airspace can be neglected. If radiation into air is to be modelled, or if an infinite or semi-infinite medium such as the ground is required, it is necessary to minimise the effect of reflections from the boundaries. For a train tunnel, where distances to be modelled are small compared with the length of the train, the z-axis boundaries are dealt with by creating a model exactly one rail vehicle (or unit of several coupled rail vehicles) in length, and then connecting the ends of the model together to create an infinitely long train. This is done by copying the cell displacements and velocities from one end of the model to the other end at the end of each time-step.

A.2.2 For the other boundaries in the x- and y-axes, the potential problem of spurious reflections from model boundaries is overcome by the use of an impedance matching technique. This effectively assigns to the cells which are required to be non-reflective on the boundaries of the model the properties of a massless viscous damper such that:

$$\frac{\eta K''}{\omega} = -\left( \rho c + \frac{D(\xi_o - \xi_{-1})}{\rho \Delta x v_0} \Delta t \right)$$

A.2.3 where $\eta$ is the loss factor (dimensionless), *K''* is the imaginary part of a complex spring stiffness in which the real part is zero, $\omega$ the angular frequency, $\rho c$ the characteristic impedance of the medium, $\xi_0$ and $\xi_{-1}$ are the displacements of cell points 0 and –1 where the boundary is at cell 0,

$\rho$ is the density of the cell contents and $v_0$ is the velocity of cell 0. Over 95% absorption is achieved across the spectrum.

## A.3 INPUT DATA

A.3.1 The only input data required for the model are the masses of each cell, plus the shear modulus and the compression modulus, and the loss factor. Otherwise, all secondary parameters such as wave speeds, impedances etc. are automatically generated by the finite difference algorithm. The only other input relates to methods of approximating actual structure shapes using the orthogonal grid.

A.3.2 The output of the model consists of a file containing the displacement and/or velocity of one or more selected cells.

A.3.3 The time steps used are of the order of 30 to 60 microseconds, and the model is run for either 16384 or 32768 steps to give a signal length of just under 1 second.

A.3.4 The resulting discrete time series can then be subjected to discrete fourier transformation to yield frequency spectra.

A.3.5 Note that, whereas in the acoustical analogy, the impedance of air varies little (except close to sources such as points), so that in most cases power is proportional to velocity squared, in elastic media, velocity transfer functions do not directly convey information about power transmission, and velocity at the receiver, in a low impedance medium, can be higher than velocity near the source, in a high impedance medium, even when there are power losses between the source and the receiver.

## A.4 VALIDATION

A.4.1 The finite difference algorithm is validated by creating models of structures for which algebraic solutions are available and comparing the eigenfrequencies and decay rates. For Timoshenko beams, plates, thin and thick cylinders the eigenfrequencies are correctly predicted.